

Statistical models diagnostics for earthquakes data

Analisi diagnostica di modelli statistici per dati sismici

Giada Adelfio ¹

Dipartimento di Scienze Statistiche e Matematiche “S. Vianelli”
Università degli Studi di Palermo, e-mail: adelfio@dssm.unipa.it

Riassunto: Si presenta l'utilizzo di un metodo di diagnostica per processi di punto spazio-temporali, volto all'interpretazione di modelli generalmente impiegati per descrivere e prevedere il verificarsi di eventi sismici in aree fissate.

Il metodo diagnostico consente di individuare le proprietà del secondo-ordine, quali la dipendenza di lungo termine e il comportamento frattale, presenti nei dati e non descritte da un particolare modello adattato.

Keywords: Point processes, residuals, second-order statistics, earthquakes occurrence.

1. Introduction

Point processes and probability theory allow to study in depth several aspects of seismological research, such as the definition of models describing space-time occurrence of seismic events, the simulation of seismic catalogs to better understand the real process, the definition of predictive strategies for the assessment of seismic risk.

A more realistic description of seismic events often requires the relaxation of the assumption of statistical independence of earthquakes. Therefore, since second-order properties of point processes, together with models that account for the complexity of data, may have a relevant role in the study and the comprehension of the seismic process and its realization, more complex models than the stationary Poisson process and features like self-similarity, long-range dependence and fractal dimension, are reviewed.

To test the goodness of fitting of models describing earthquakes occurrence, the introduction of methods for residuals analysis is required. Here a diagnostic method is proposed and applied to real data; it is based on a transformed version of second-order statistics, weighting the original point process by the inverse of its conditional intensity function, so that features such as clustering and inhibition can easily be interpreted. Indeed, the null hypothesis model is an arbitrary conditional intensity model, rather than a stationary Poisson process.

2. Diagnostic analysis for seismic occurrence models

To interpret the goodness of fit of models to spatial-temporal point patterns, a new diagnostics tool is here provided. It is based on the definition of a weighted version of second-order statistics (such as autocorrelation, K -function, spectrum, fractal dimension

¹Address of correspondence: Dipartimento di Scienze Statistiche e Matematiche “S. Vianelli”, viale delle Scienze ed. 13 - 90128, Palermo, Italy

and R/S statistic), calculated after the weighting of the whole set of points by the inverse of its conditional intensity function. The proposed residuals analysis deals with the estimation of weighted second-order statistics and their interpretation to detect features like clustering and long-range dependence of data not accounted for by a wrong fitted model. In particular, applications to real data are provided, based on the interpretation of the residual measures after estimating and fitting a particular space-time process Epidemic Type Aftershocks-Sequences (ETAS model; Ogata (1998)) to seismic events.

ETAS model is a self-exciting point process, describing the occurrence of earthquakes in a region during a period of time through a branching structure.

ETAS model is completely characterized by its conditional intensity function, which provides a quantitative evaluation of future seismic activity and is proportional to the probability that an event with magnitude m (a logarithmic measure of the earthquake strength) will take place at time t , in the epicenter of coordinates (x, y) , that is a position on the surface of the Earth expressed in latitude and longitude. It is defined as the sum of a term describing spontaneous activity (background) and one relative to the induced seismicity.

For example, assume that data are simulated from an ETAS model with a power law induced density and that a model with an exponential one is fitted. In this case, higher values in the autocorrelation function of the weighted points at small lags may become visible, indicating clustering in data not taken into account by the fitted model.

Suppose data come from the ETAS model. Usually, background spatial component is estimated by isotropic bivariate kernel, just using greater events (Zhuang *et al.*, 2002). This approach may provide a bad fitting, especially when points are strictly aligned along faults. Indeed, fitting an isotropic background for such a structure of data, the proposed residuals analysis (by using a weighted version of the K -function (Ripley, 1976) or the correlation dimension (Grassberger and Procaccia, 1983)) amplifies the features of the data that deviate from the model, because of the very smoothed intensity function estimated in this case. Graphical interpretations of the residuals measures give indications for the use of anisotropic kernel densities.

Weighted second-order statistics directly apply to data without assuming homogeneity or transforming data into residuals, eliminating the sampling variability due to the use of a transforming procedure (Schoenberg, 2003); this method can be applied to processes of any dimension, provided that statistics exist and gives to second-order statistics a primary role in the diagnostics aim, to interpret dependence and attractive/inhibitive features of observed point patterns.

References

- Grassberger P. and Procaccia I. (1983) Measuring the strangeness of a strange attractor, *Physica D*, 9, 189–208.
- Ogata Y. (1998) Space-time point-process models for earthquake occurrences, *Annals of the Institute of Statistical Mathematics*, 50, 2, 379–402.
- Ripley B.D. (1976) The second-order analysis of stationary point processes, *Journal of Applied Probability*, 13, 2, 255–266.
- Schoenberg F.P. (2003) Multi-dimensional residual analysis of point process models for earthquake occurrences, *Journal American Statistical Association*, 98, 464, 789–795.
- Zhuang J., Ogata Y. and Vere-Jones D. (2002) Stochastic declustering of space-time earthquake occurrences, *Journal of the American Statistical Association*, 97, 458, 369–379.